CLEARSY
SYSTEM ENGINEERING

Training for THE B METHOD

Level 1

overview

- *introduction to B*
  - a formal method…
    …with proofs
  - the usage of B
  - foundations
  - benefits
- *concepts of B*
  - B modules
  - B components
    - abstract machines
    - refinements
    - implementations
  - B projects
- *the B language*
  - predicate logic
  - set theory
  - substitutions
  - data typing
  - form of components
  - Modular decomposition

introducing B

- *a formal method …*
  - specification method based on a mathematical formalism to build models
- *… with proofs*
  - to prove that a model is consistent (in every possible case)
- *used for:*
  - systems specification
  - software development

B and Atelier B: a formal method for

- *B for Systems*
  - Goal: help to understand, specify, design, verify a system development
    - not a method to create a system, but to check it
    - requires contacts with the system creators to deeply understand the system
  - a B-System model formalizes:
    - the system (hardware and software)
    - its environment (other systems, infrastructure, procedures handled by operators)
  - covers functional logical angle of the system, not digital calculus, not real-time requirements

B and Atelier B: a formal method for

B for developing (safety-critical) Software

- Goal: to develop a code that complies with its specification and to be sure of it (to know exactly what is proved)
- a B-Software model formalizes the software itself, through a modules break down
- covers a subpart of the software with functional logical procedures, only for one task or thread, not low-level Operating System features, no direct input/output

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B-Software: Industrial References

- *KVB: Alstom*
  Automatic Train Protection for the French railway company (SNCF), installed on 6,000 trains since 1993
  60,000 lines of B; 10,000 proofs; 22,000 lines of Ada

- *SAET METEOR: Siemens Transportation Systems*
  Automatic Train Control: new driverless metro line 14 in Paris (RATP), 1998. 3 safety-critical software parts: onboard, section, line
  107,000 lines of B; 29,000 proofs; 87,000 lines of Ada

- *Roissy VAL: ClearSy (for STS)*
  Section Automatic Pilot: light driverless shuttle for Paris-Roissy airport (ADP), 2006
  28,000+155,000 lines of B; 43,000 proofs; 158,000 lines of Ada

B-System: Industrial References

- *Peugeot Automobiles*
  - Model of the functioning of subsystems (lightings, airbags, engine, …) for Peugeot aftersales service
  - Goal: Understanding precisely the functioning of cars to build tools to diagnose breakdowns
- *RATP (Paris Transportation)*
  - Model of automatic platform doors to equip an existing metro line
  - Goal: Verifying consistency of System Specification

B-System References

- *EADS*
  - Model of tasks scheduling of the software controlling stage separation of Ariane rocket
- *Study of a Communication Protocol*
  - Proof that the algorithm of a communication protocol complies with its requirements
- *INRS (French Institute for Workers Safety)*
  - Model of a mechanical press complying with safety requirements (protection of the hands of the press operator)
  - Building the software specification of the press controller

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basic concepts

- B is a method for specification (and possibly for programming)
  - B formalized system properties, static description, dynamic description
- B is based on a mathematical language
  - predicates, Booleans, sets, relations, functions
- B is structured
  - the notion of module
  - the notion of refinement
- B is a framework for development, validated by **proofs**
  - proof validation: systematic debugging

B structuring

- *the notion of modules*
  - to break down a large system or software into smaller parts
  - a module has a specification, where to formalize:
    - system properties
    - static description of requirements
    - dynamic description of requirements

B structuring

- *the notion of refinement*
  - a module specification is refined: it is reexpressed with more information:
    - adding some requirements
    - refining abstract notions with more concrete notions (design choice)
    - for B-software, getting to implementable code level
  - a refinement must be consistent with its specification (this should be proved)
  - a refinement may also be refined (refinement column)
  - for B-software, the final refinement is called the implementation

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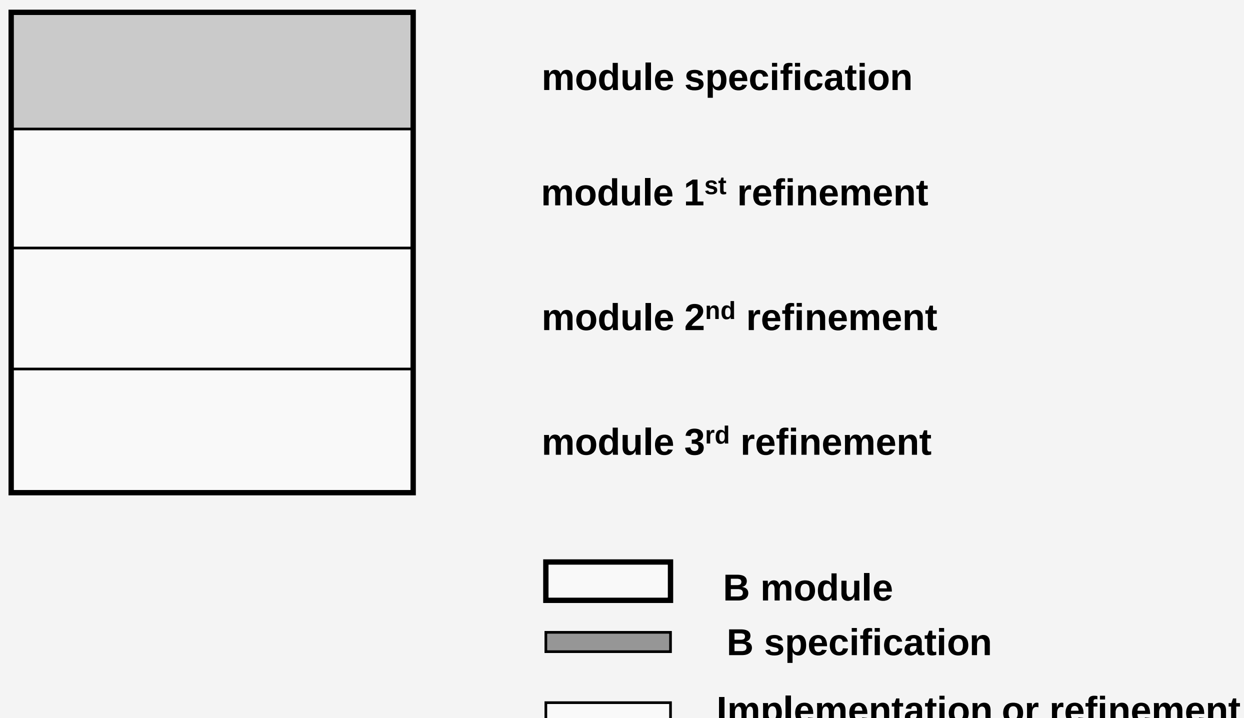
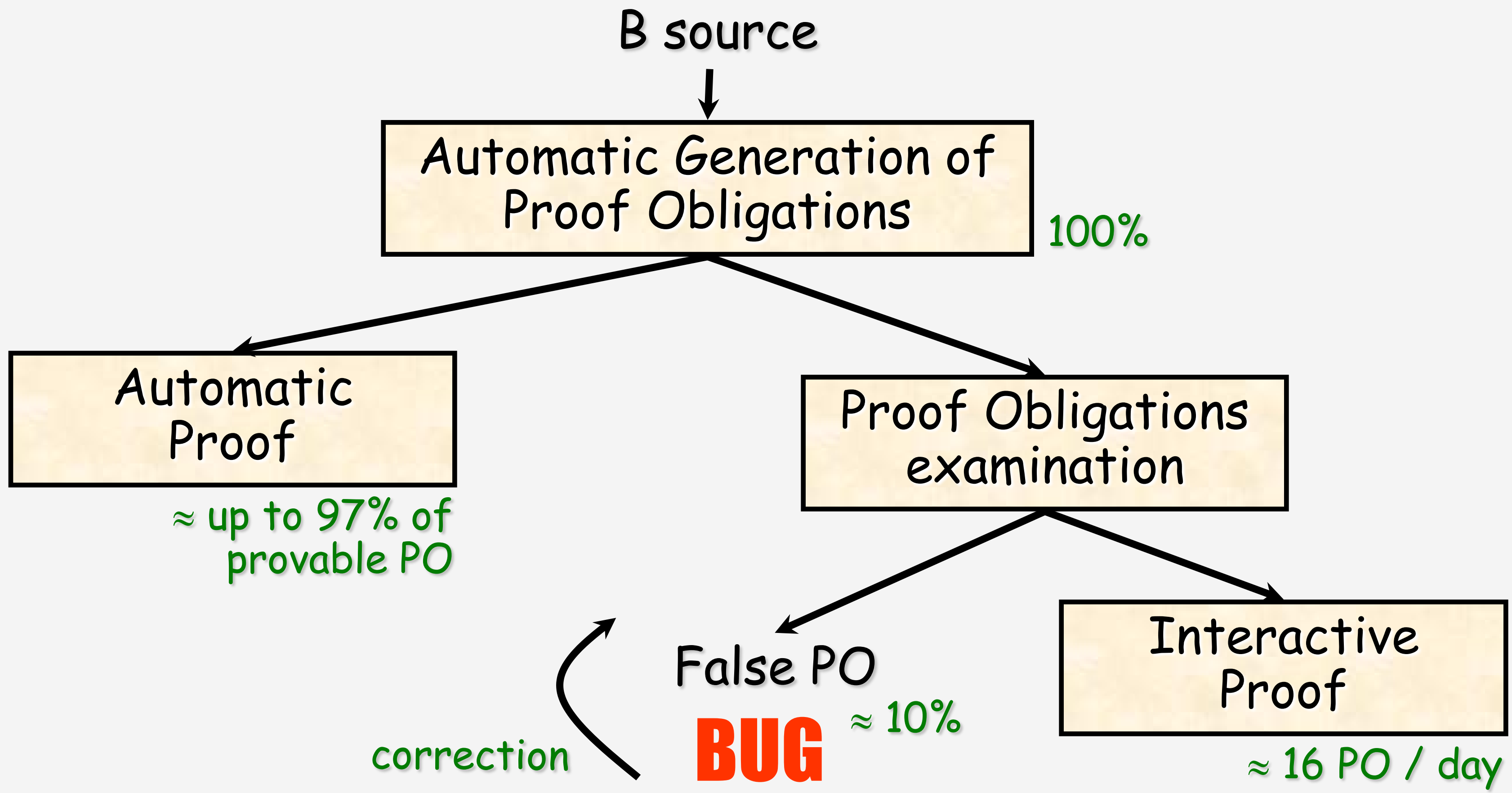
B structuring



validation by the proof



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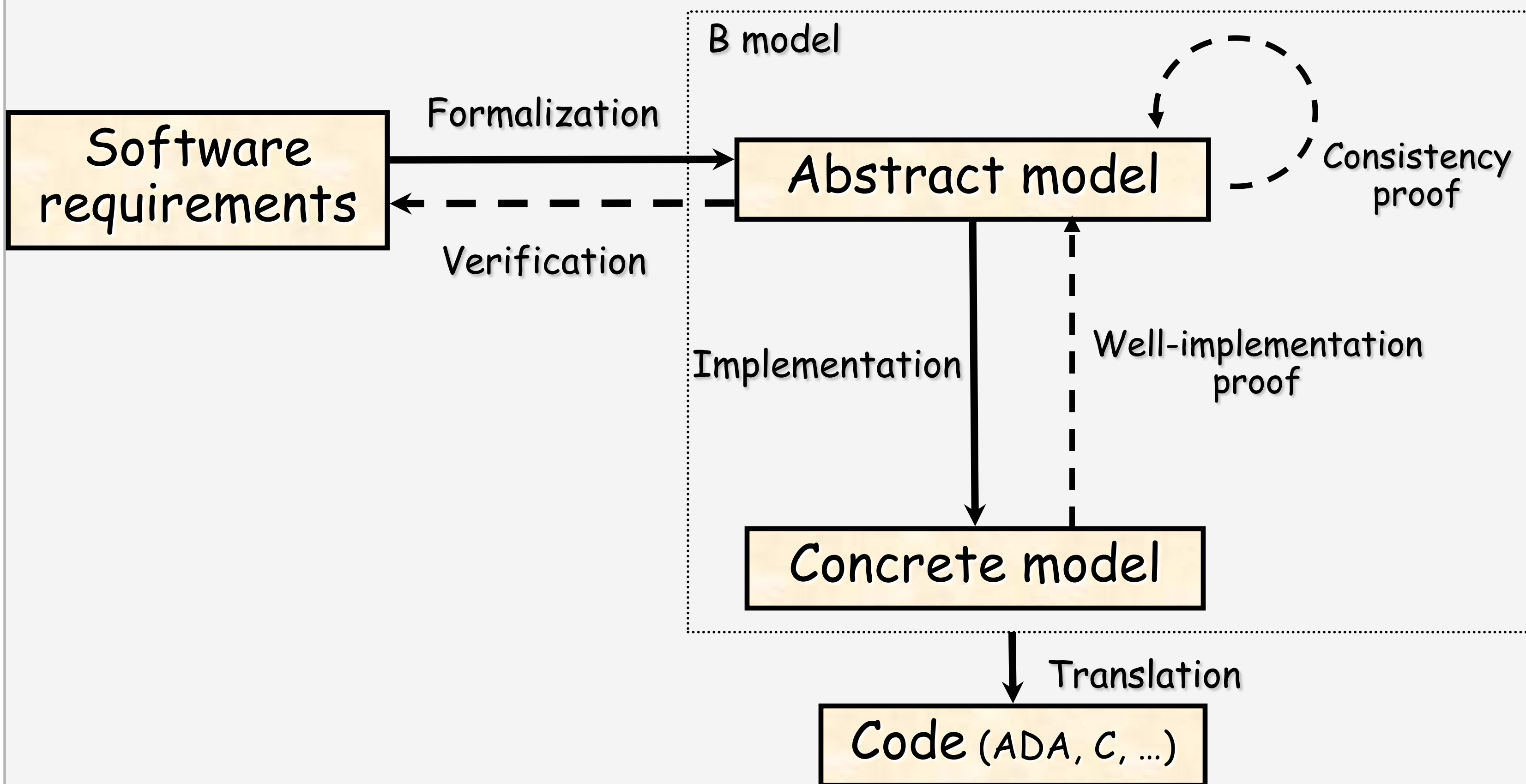
B-Software



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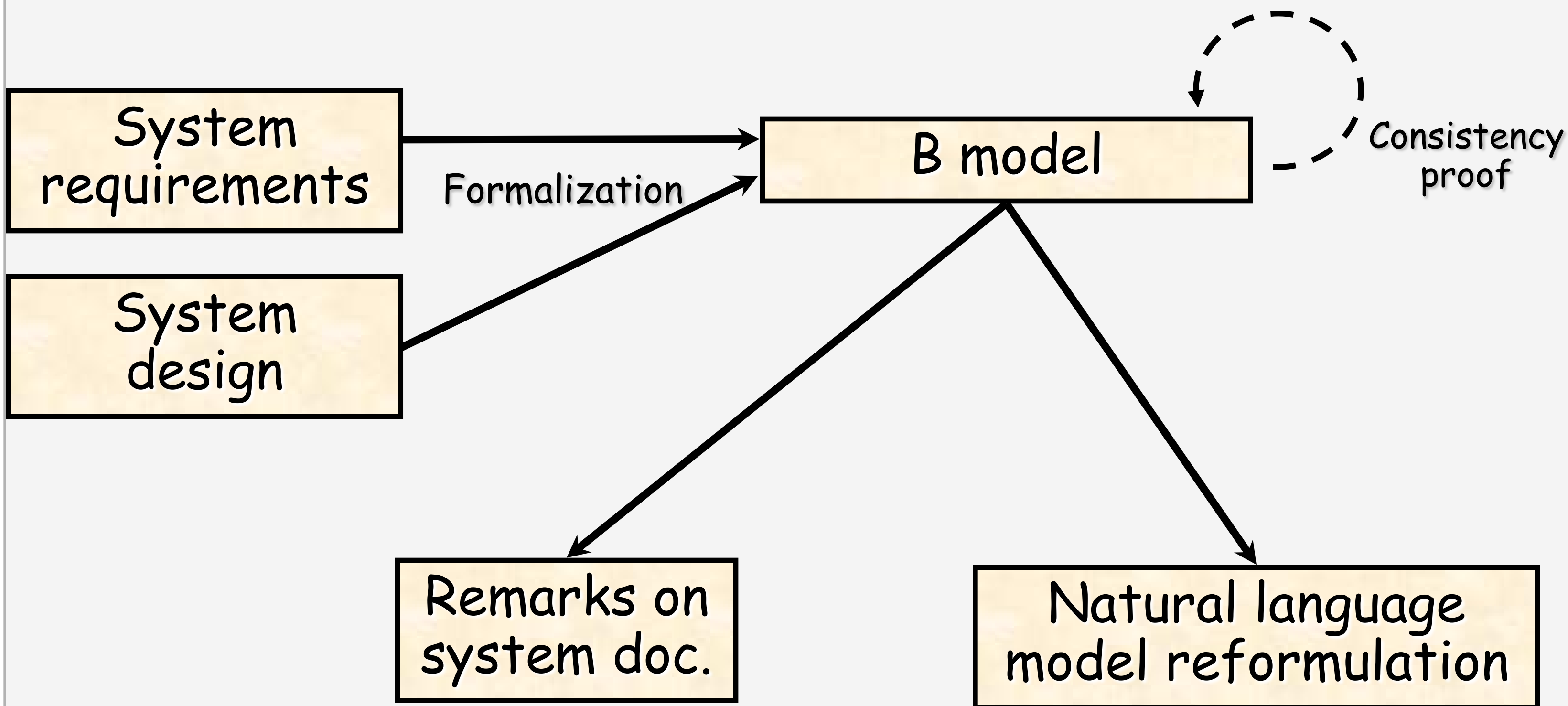
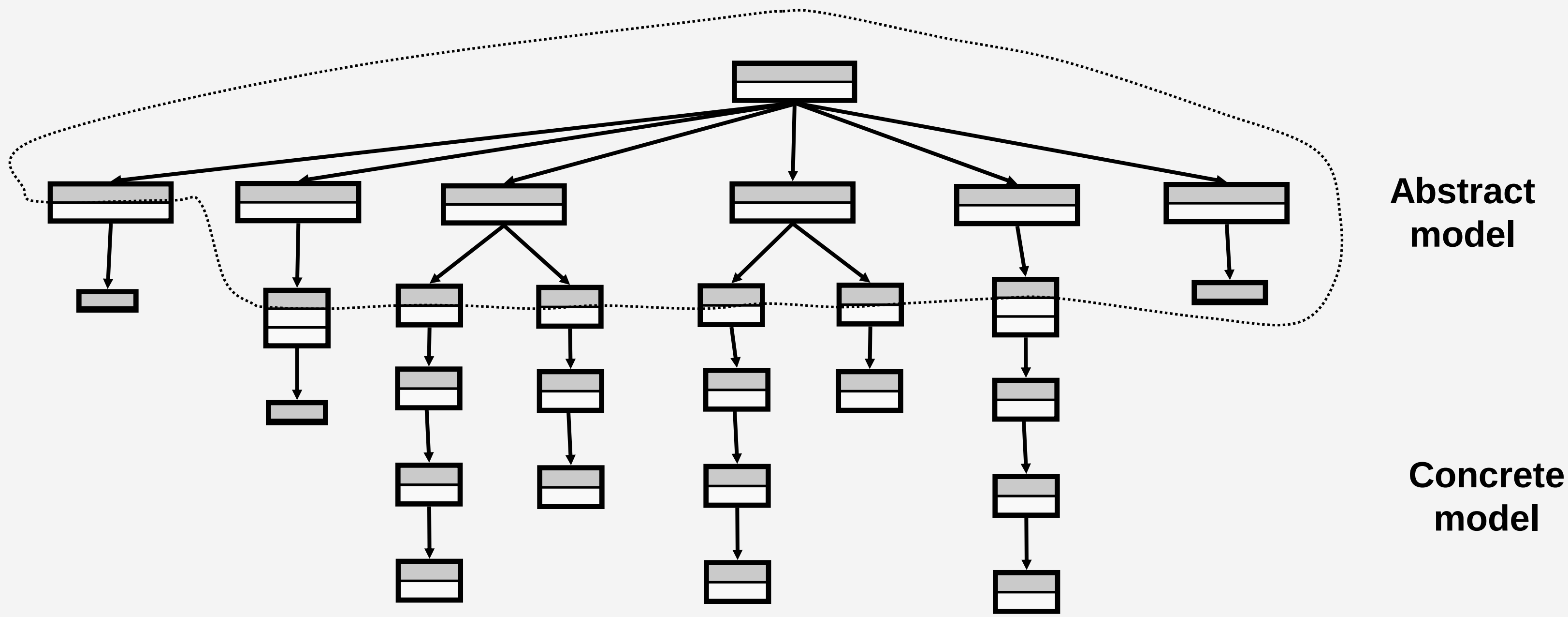
B-System


System requirements
System design
Formalization
B model
Consistency proof
Remarks on system doc.
Natural language model reformulation

B-Software structure



B module

B specification

Implementation or refinement

Decomposition link

benefits of B-Software

- *The Abstract Model*
  - Requirements are formalized into B specifications module by module
    Non-formal and formal specification are very close (they both express **what** the software should do) to minimize errors
  - Some software properties are formalized into B
    They strengthen the B model, since we must **prove** that they remain true when the modules are put together
- *The Concrete Model*
  - We must **prove** that the concrete model complies with its specification (the abstract model)

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benefits of B-Software

- *The whole Model*
  - ➔ NO classic programming error in the code (overflow, division by 0, out of range index, infinite loop, aliases)
  - ➔ A healthy program architecture
  - ➔ Unit Test are no longer used
  - ➔ Early detection of errors
  - ➔ These benefits remain even after some modifications/evolutions

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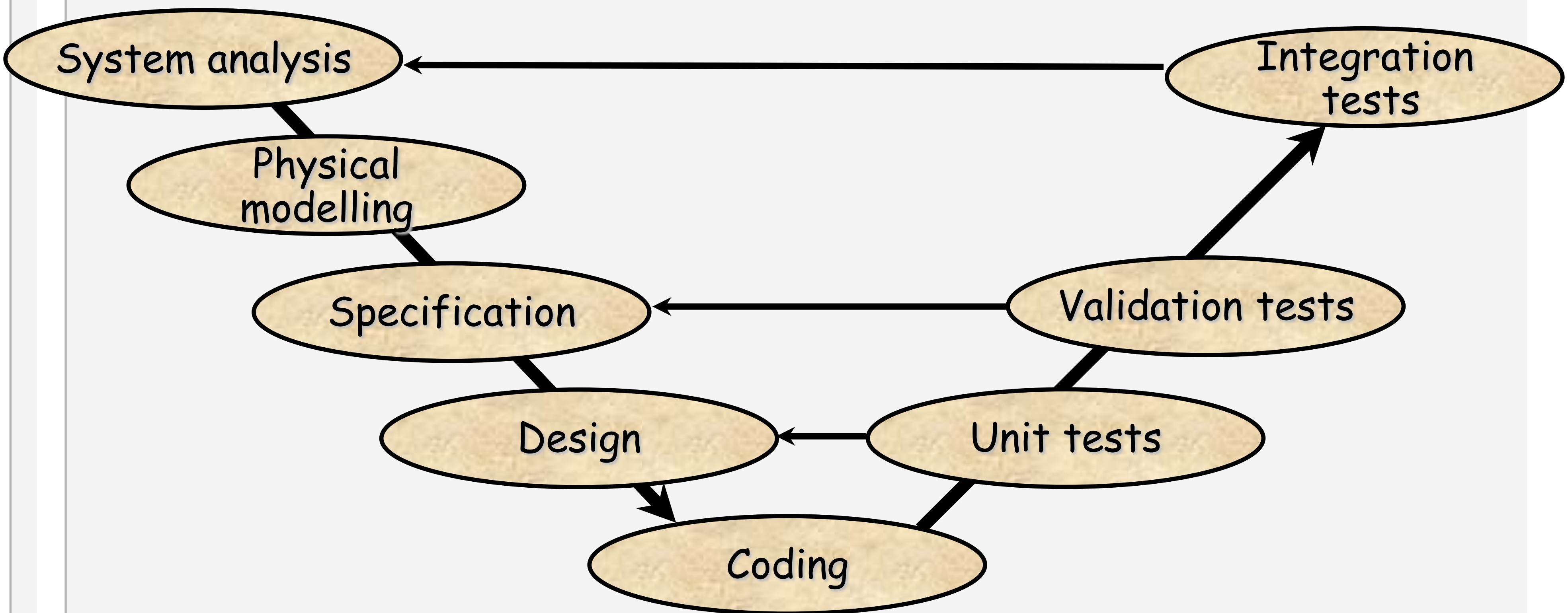
traditional development cycle



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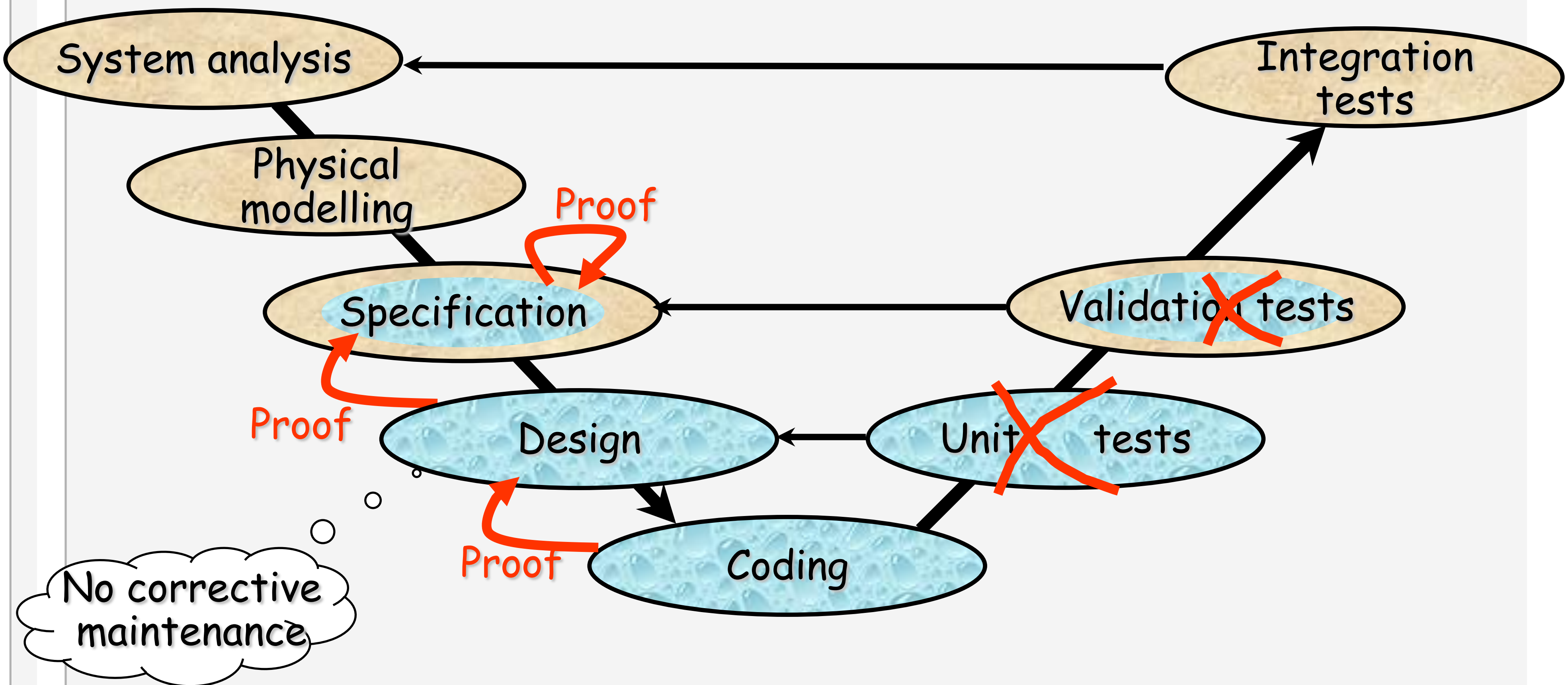
the B development cycle



comparison with other languages

- *Assembly Language*
  - → no static control
- *C Language*
  - → limited **static** controls (e.g. typing for data size)
- *Ada Language*
  - → extended **static** controls (e.g. strong typing)
- *B Method*
  - → static controls + controlling the program **meaning**
    (by **proving** that the B specification is consistent **and**
    by **proving** that the B code complies with its specification)

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benefits of B-System

- *B-System model*
  - The bottom line is to deeply understand the system through the B model construction
  - A work in cooperation with system creators
  - Early detections of errors, at system design level, producing better Software Specification
- *Remarks on the system*
  - most questions on the system arise during creation of the B model
  - a few inconsistencies may also be detected through model proof (since the model should be consistent in every possible case)
- *Produces*
  - interesting remarks on the system
  - a natural language reformulation of the model giving a sharp, concise and highly structured system description

The Tools

- *Atelier B (ClearSy)*
  - created to develop industrial B-Software projects
  - a set of tools integrated into a project manager tool
    - static checkers
    - automatic proof obligation generator
    - automatic provers and interactive prover
    - code translators: Ada, C, …
  - it is also used for B-System
- *B4free (www.b4free.com)*
  - free but restricted to academic users and owners of Atelier B
  - the core tools of Atelier B + a new x-emacs interface
- *Rodin platform (September 2007)*
  - a new open platform dedicated to B-System (in construction)

ClearSy: activities related to B

- uses B-System internally to help understand, specify, verify a system development
- uses B-Software internally to develop safety-critical software (and also to finish up proof or validate proof of B-software projects)
- is part of the B community and tries to create useful processes based on B
- training sessions for the B Language and Atelier B
- development, distribution and support of Atelier B

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B in education

- *30 universities/research labs currently active*
- *300 graduates per year with some experience*

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conclusion

- *B is a language*
- *B is a development method*
- *B ensures* correct *systems and software*

- *B is used successfully by industry*
- *B is supported by a tool:* ***Atelier B***
- *B brings concrete benefits to its users*

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concepts of B: modules

- a B module corresponds to a subsystem model (eventually software)
- each B module manages its own data space : "data encapsulation"

  cf. classes (object-oriented languages)
  abstract data types
  packages (ADA)
- a fully developed B module consists of several **B components**
  - an **abstract machine** (the module specification)
  - some possible **refinements** (of its specification)
  - an **implementation** (final refinement: B0 code)
- these components are maintained within a single **B project**

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concepts of B: components

- **static aspect**
  - definition of the subsystem state space: *sets*, *constants*, *variables*
  - definition of static properties for its state variables: *invariant*
- **dynamic aspect**
  - definition of the initialisation phase (for the state variables)
  - definition of operations for querying or modifying the state
- **proof obligations**
  - the static properties are consistent
  - they are *established* by the initialisation
  - they are *preserved* by all operations

concepts of B: abstract machines

- an abstract machine is the **formal specification** of a software module
- it defines a **mathematical model** of the subsystem concerned
  - an abstract description of its state space and possible initial states
  - an abstract description of operations to query or modify the state
- this model establishes the **external interface** for that module
  - every implementation will conform to its specification
  - this guarantee is assured by *proves* that has to be done during the formal development process

concepts of B: an abstract machine

general form

```
MACHINE
        machine name
SETS
        set names
CONSTANTS
        constant names
PROPERTIES
        predicate
VARIABLES
        variable names
INVARIANT
        predicate
INITIALISATION
        substitution
OPERATIONS
        operation definitions
END
```

static aspect (MACHINE … INVARIANT)

dynamic aspect (INITIALISATION … OPERATIONS)

concepts of B: refinements

- components that refine an abstract machine (or its most recent refinement)
- they add new properties to the previous math. model (more detailed properties) and make it more concrete
  - data refinement
    introduction of new variables to represent the state variables for the refined component, with their *linking invariant*
  - algorithmic refinement
    transformation of the operations for the refined component
- correctness of development
  - each refinement *has to* preserve the properties of the component it refines

concepts of B : refinements

an intermediate refinement

general form

```
REFINEMENT
        machine name_n
REFINES
        machine name
...

VARIABLES
        variable names
INVARIANT
        predicate
INITIALISATION
        initialisation refinement
OPERATIONS
        operation refinements
END
```

data for the refined component (sets and constants are preserved)

new variables with their own properties + linking invariant

it is not possible to introduce new operations here

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concepts of B: implementations

a final *refinement containing B0:*
the B code, that can be executed

general form

IMPLEMENTATION
machine name_n
REFINES
machine name
VALUES
valuations
CONCRETE_VARIABLES
variable names
INVARIANT
predicate
INITIALISATION
initialisation implementation
OPERATIONS
operation implementations
END

values for fixed sets
and constants

implementation variables
with
their invariant properties
+ linking invariant

concepts of B: projects

- a B project is a set of linked B modules

  each module is formed of components: an abstract machine

  (its specification), possibly some refinements and an implementation
- the principal dependencies links between modules are
  - **IMPORTS** links (forming a *modular decomposition* tree)
  - **SEES** links (read only transversal visibility)
- sub-projects may be grouped into *libraries*
- a B project supports formal development of software (translation in Ada, C, C++)
  - software developed in B may integrate or may be integrated with traditionally developed code

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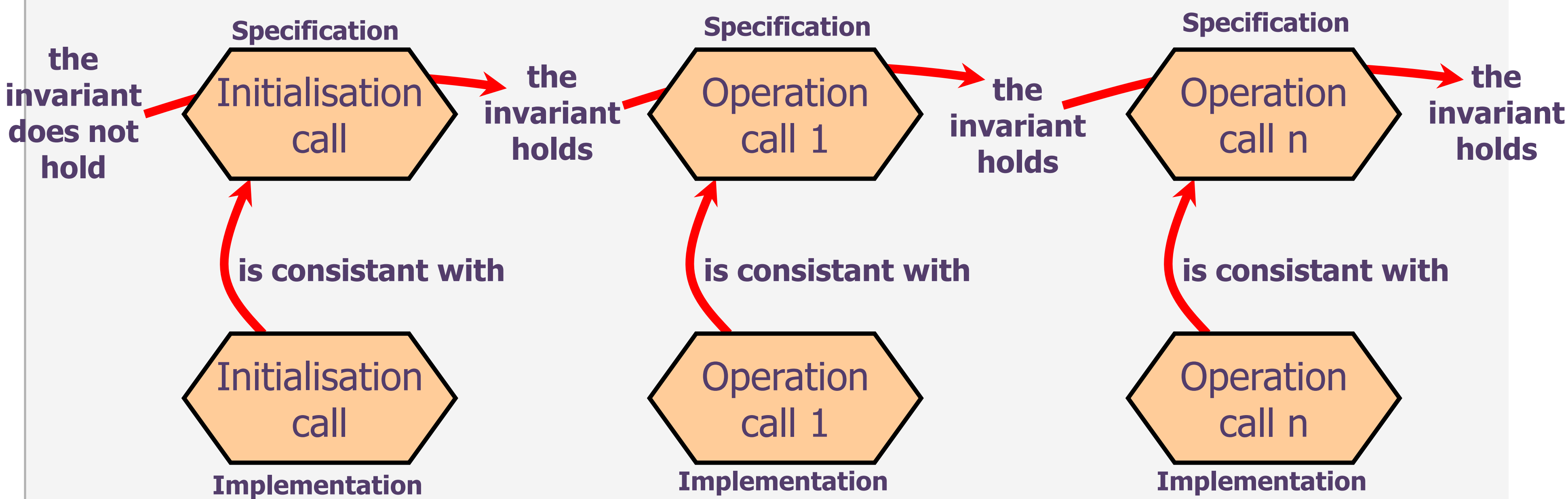
Concepts of B: what is proved?



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the B language

- *order of presentation*
  - **predicate** logic
  - set theory (B **expressions**)
  - **substitutions**
  - data typing
  - form of components
  - modular decomposition

the B language: predicate logic

- *Predicates*
  - the way to express properties
  - a predicate is a logical formula, which may or may not *hold* *(is true or is false)*
  - equations, inequalities and membership of a set are simple predicates

    e.g. $x = 3$

    $5 < 2$

    $x : \{1, 2, 3\}$
  - Simple predicates may be combined by negation, conjunction or disjunction

    e.g. $x + y = 0 \;\&\; x < y$

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the B language: predicate logic

on paper	at the keyboard
n	not
&	&
o	or
y	=>
e	<=>
!	!
#	#
.	.

- *propositions*

n P	negation of P (logical NOT)
P & Q	conjunction of P and Q (logical AND)
P o Q	disjunction of P and Q (logical OR)
P y Q	logical implication: n P o Q
P e Q	logical equivalence: P y Q & Q y P

- *quantified predicates*

! x . (P_x y Q_x)	universal quantification
# x . (P_x)	existential quantification: n (! x . (n P_x))

the B language: predicate logic

on paper	at the keyboard
=	=
d	/=
<	<
<	<=
>	>
>	>=

- *equality predicates*
  - let x and y be two expressions

$x = y$	x equal to y
$x \neq y$	non equality: n ($x = y$)

- *inequality predicates*
  - let x and y be two integer expressions

$x < y$	x strictly less than y
x < y	x less than or equal to y
$x > y$	x strictly greater than y
x > y	x greater than or equal to y

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the B language: predicate logic

on paper	at the keyboard
:	:
/	/:
(	<:
-	/<:
è	<<:
_	/<<:

set predicates

- let *x* be an element, and let *X* and *Y* be two sets

x : *X*	membership: *x* is an element of *X*
x / *X*	non membership
X (*Y*	inclusion: *X* is a subset of *Y*
X - *Y*	non inclusion
X è *Y*	strict-inclusion: *X* è *Y* e *X* (*Y* & *X* d *Y*
X _ *Y*	non strict-inclusion

the B language

- *set theory (B expressions)*
  - sets
  - subsets
  - Boolean set
  - numeric sets
  - sets of maplets
  - relations
  - functions
  - sequences

the B language: set theory

on paper	at the keyboard
0	{ }

- *explicit sets*
  - empty set: 0
  - finite set defined in extension: $\{ x_1, x_2, \ldots, x_n \}$
    e.g. {a, b, c}
    $\{1, x+1, y-2\}$
  - set of integers between *x* and *y* (interval):
    $z : (x .. y) \;\; e \;\; z > x \;\&\; z < y$
    e.g. 1..3 = ...
    3..2 = ...
- *sets defined in comprehension*
  - $\{ x \mid P_x \}$ subset of *X* such that the predicate *P* holds
    e.g. $\{ x \mid x : 1..5 \;\&\; x \bmod 2 = 0 \} = \ldots$

the B language: set theory

on paper	at the keyboard
u	\/
i	/\

set expressions

- let *X* and *Y* be two sets

X u *Y* union of *X* and *Y* :

z : (*X* u *Y*) e z : *X* o z : *Y*

e.g. {1, 3, 5} u 1..3 = ...

X i *Y* intersection of *X* and *Y* :

z : (*X* i *Y*) e z : *X* & z : *Y*

e.g. {1, 3, 5} i 1..3 = ...

X - *Y* set deferrence of *X* and *Y* :

z : (*X* - *Y*) e z : *X* & z / *Y*

e.g. {1, 3, 5} - 1..3 = ...

1..3 - {1, 3, 5} = ...

the B language: set theory

on paper	at the keyboard
P	POW
P_1	POW1
F	FIN
F_1	FIN1

subset types

- let X be a set

$\mathrm{P}(X)$ the set (type) of subsets of X:

$x : \mathrm{P}(X) \quad \mathrm{e} \quad x \, (\, X$

$\mathrm{P}_1(X)$ the set (type) of non-empty subsets of X:

$\mathrm{P}_1(X) \quad = \quad \mathrm{P}(X) - \{0\}$

e.g. $\mathrm{P}(\{1,2,3\}) = \ldots$

$\mathrm{P}_1(\{1,2,3\}) = \ldots$

$\mathrm{F}(X)$ the set (type) of *finite* subsets of X

$\mathrm{F}_1(X)$ the set (type) of finite non-empty subsets of X:

$\mathrm{F}_1(X) \quad = \quad \mathrm{F}(X) - \{0\}$

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the B language: set theory

- *Boolean constants*
  - → TRUE, FALSE predefined constants
- *Boolean expressions*
  - → BOOL predefined Boolean set

    BOOL = {TRUE, FALSE}
- *Boolean expressions*
  - → let *P* be a predicate

    the value of bool (*P*) is TRUE if *P* holds, otherwise FALSE

    e.g. bool(*P*) = bool(*Q*) e ...

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the B language: set theory

- *numeric expressions*

on paper	at the keyboard
*	*
x^{n}	$x ** n$

- card (X) cardinal of X: its number of elements

  e.g. card ({1,3,5}) = ...

- max (X), min (X) maximum, minimum of X

  e.g. max ({1,3,5}) = ...

  min ({1,3,5}) = ...

- let x and y be integers, m be a non-zero natural number and n be a natural number

pred (x), succ (x)	predecessor, successor
$x + y$, $x - y$	addition, subtraction
$x * y$, x / m	multiplication, integer division
x mod m	modulo
x^{n}	power

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the B language: set theory

on paper	at the keyboard
$\mathbb{Z}$	INTEGER
$\mathbb{N}$	NATURAL
$\mathbb{N}_1$	NATURAL1

- *Sets of integers*

  $\mathbb{Z}$ set of relative integers (>0 and <0)

  $\mathbb{N}$ set of natural integers (> 0)

  $\mathbb{N}_1$ set of positive natural integers (> 0)

- *numeric constants*

  MAXINT the largest implementable relative integer

  MININT the smallest implementable relative integer

  INT predefined set of implementable relative integers
  INT = MININT .. MAXINT

  NAT predefined set of implementable natural numbers
  NAT = 0 .. MAXINT

  NAT_1 predefined set of strictly positive implementable natural numbers
  NAT_1 = 1 .. MAXINT

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the B language: set theory

on paper	at the keyboard
m	|->
*	*

maplet expression

- let x and y be two elements

x, y ordered couple or

x m y 'maplet' (x associated to y)

x m y = x, y

cartesian product types

- let X and Y be two sets

X * Y cartesian product of X and Y

the set (type) of maplets x, y such that x : X et y : Y

e.g. {0, 1} * {a, b, c} = ...

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the B language: set theory

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	1	<->

- *relations*
  - definition: a **relation** from a source set X into a target set Y is a subset of the cartesian product $X * Y$, that is a set of maplets where the first element belongs to X and the second to Y
  - consequence: set expressions may also be applied to relations
  - such a relation is denoted: $R : X \; 1 \; Y$
- *relation types*
  - the set of relations from X into Y:

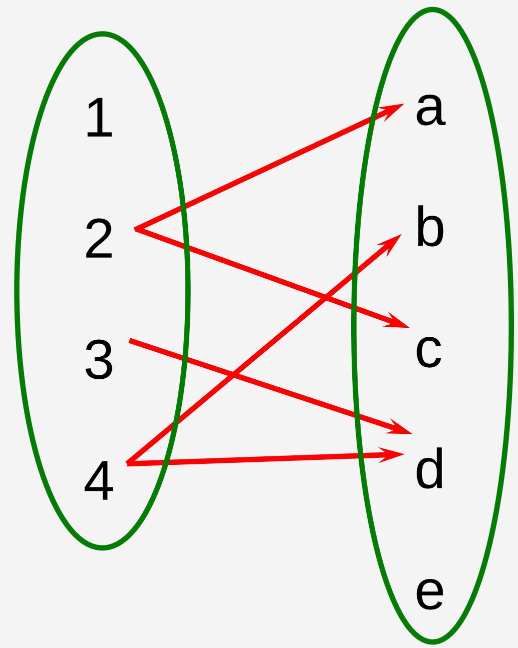
$$X \; 1 \; Y = \mathrm{P}(X * Y)$$

the B language: set theory

- *Explicit relations*

  e.g. : 0 (empty relation)

  {2 m a,
  2 m c,
  3 m d,
  4 m b,
  4 m d}



the B language: set theory

- *relational operations*
  - let R be a relation from X into Y

dom (R) domain of the relation R (a subset of X)

$x \in \mathrm{dom}\,(R) \Leftrightarrow \exists y \cdot (y \in Y \land x \mapsto y \in R)$

e.g. dom ({2 $\mapsto$ a, 2 $\mapsto$ c, 3 $\mapsto$ d, 4 $\mapsto$ b, 4 $\mapsto$ d}) = ...

ran (R) codomain or *range* of the relation R (a subset of Y)

$y \in \mathrm{ran}\,(R) \Leftrightarrow \exists x \cdot (x \in X \land x \mapsto y \in R)$

e.g. ran ({2 $\mapsto$ a, 2 $\mapsto$ c, 3 $\mapsto$ d, 4 $\mapsto$ b, 4 $\mapsto$ d}) = ...

  - let R be a relation from X into Y, and S be a subset of X

$R[S]$ image of the set S through the relation R (a subset of Y)

$y \in R[S] \Leftrightarrow \exists s \cdot (s \in S \land s \mapsto y \in R)$

e.g. {2 $\mapsto$ a, 2 $\mapsto$ c, 3 $\mapsto$ d, 4 $\mapsto$ b, 4 $\mapsto$ d} [{1,2,3}] = ...

the B language: set theory

on paper	at the keyboard
R^{-1}	$R\sim$

relational expressions

- let X be a set

id (X) — identity on X (the set of maplets $x \mapsto x$, for $x \in X$):

$x \mapsto x \in \mathrm{id}(X) \Leftrightarrow x \in X$

e.g. id ({a, b, c}) = ...

- let R be a relation from X into Y

R^{-1} — converse relation (inverse maplets, from Y into X):

$y \mapsto x \in R^{-1} \Leftrightarrow x \mapsto y \in R$

e.g. $\{2 \mapsto a, 2 \mapsto c, 3 \mapsto d, 4 \mapsto b, 4 \mapsto d\}^{-1}$ = ...

the B language: set theory

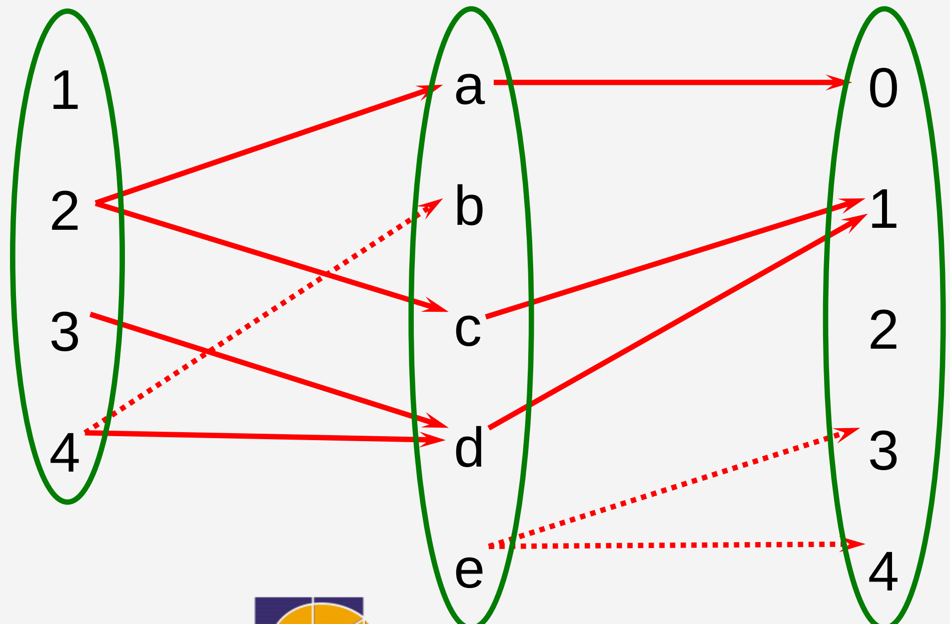
- *composition of relational expressions*

composition of the relations R_1 and R_2: $R_1 \,;\, R_2$

$x \mathrm{~m~} z : (R_1 \,;\, R_2) \mathrm{~e~} \# y. (y : Y \,\&\, x \mathrm{~m~} y : R_1 \,\&\, y \mathrm{~m~} z : R_2)$

e.g. {2 m a, 2 m c, 3 m d, 4 m b, 4 m d} ; {a m 0, c m 1, d m 1, e m 3, e m 4}

= ...



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the B language: set theory

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r	<|
R	|>

filtering relational expressions

- *S* r *R* restriction to the set *S* over the domain of relation *R* (keeping only maplets with first elements belonging to *S*)

  S r *R* = id (*S*) ; *R*

  e.g. {1, 2, 3} r {2 m a, 2 m c, 3 m d, 4 m b, 4 m d} = ...

- *R* R *S* restriction to the set *S* over the codomain of relation *R* (keeping only maplets with second elements belonging to *S*)

  R R *S* = *R* ; id (*S*)

  e.g. {2 m a, 2 m c, 3 m d, 4 m b, 4 m d} R {b, c, d} = ...

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the B language: set theory

on paper	at the keyboard
a	<<|
A	|>>

filtering relational expressions (cont.)

- *S* a *R* exclusion of the set *S* from the domain of relation *R*
  (removing maplets with first elements belonging to *S*)
  S a *R* = id (dom (*R*) – *S*) ; *R*
  e.g. {1, 2, 3} a {2 m a, 2 m c, 3 m d, 4 m b, 4 m d} = ...

- *R* A *S* exclusion of the set *S* from the codomain of relation *R*
  (removing maplets with second elements belonging to *S*)
  R A *S* = *R* ; id (ran (*R*) – *S*)
  e.g. {2 m a, 2 m c, 3 md, 4 m b, 4 m d} A {b, c, d} = ...

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the B language: set theory

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- *relational expressions (cont.)*
  - ➔ let R_1 and R_2 be two relations from X into Y

$R_1 + R_2$ overloading of relation R_1 by relation R_2 to obtain the relation with maplets from R_1 where their first elements do not belong to dom (R_2), together with all maplets from R_2

$$R_1 + R_2 \;=\; (\mathrm{dom}\,(R_2)\ \mathrm{a}\ R_1)\ \mathrm{u}\ R_2$$

e.g. {0 m 1, 1 m 1} + {0 m 0} = ...

{2 m a, 2 m c, 3 m d, 4 m b, 4 m d} + {1 m a, 2 m b, 2 m c, 3 m e}

= ...

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the B language: set theory

- *functions*
  - reminder
    - definition: a **relation** from a source set X into a target set Y is a subset of the cartesian product $X * Y$, that is a set of maplets where the first element belongs to X and the second to Y
    - consequence: set operations also apply to relations
  - special case
    - definition: a **function** from a source set X into a target set Y is a relation from X into Y, such that each element of X is associated to *at most one* element of Y (but in general, the inverse of a function is not itself a function)
    - consequence: relational expressions also apply to functions

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the B language: set theory

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- *function applications*
  - let F be a function, and x be an element of dom (F)

$F\,(x)$ the (unique) value of function F for x
the associated element from ran (F), where x m $F\,(x)$: F

e.g. f = { 0 ma, 1mb, 2 ma }

f (1) = ...

- *functions defined by expressions*
  - let x be a name, X be a set and E be an expression (in x)

% x . (x : X | $E(x)$) explicit definition in the form of a %-expression
the function consisting of maplets x m $E(x)$, for x : X

e.g. plus2 = % z . (z : Z | z+2)

plus2 (1) = ...

the B language: set theory

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function types

- definition: in general, a function from the set X into the set Y is called a **partial function**
- such a function is denoted: F : X 2 Y
- F : X 2 Y e F : X 1 Y & (F $^{-1}$; F) (id (Y)
- definition: a **total function** is a function from X into Y where the domain is equal to X
- such a function is denoted: F : X 3 Y
- F : X 3 Y e F : X 2 Y & dom (F) = X

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5	>->

- *injective functions*
  - definition: a **partial injection** is a function from X into Y where each range element has one and only one antecedent
  - such a function is denoted: $F : X \; 4 \; Y$

    $F : X \; 4 \; Y \;\; e \;\; F : X \; 2 \; Y \; \& \; F^{-1} : Y \; 2 \; X$
  - definition: a **total injection** is an injection from X into Y where the domain is equal to X
  - such a function is denoted: $F : X \; 5 \; Y$

    $X \; 5 \; Y \; = \; X \; 4 \; Y \; i \; X \; 3 \; Y$

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the B language: set theory

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7	-->>

- *surjective functions*
  - definition: a **partial surjection** is a function from X into Y where the range is equal to Y
  - such a function is denoted: $F : X \; 6 \; Y$

    $F : X \; 6 \; Y \;\; e \;\; F : X \; 2 \; Y \; \& \; \mathrm{ran}\,(F) = Y$
  - definition: a **total surjection** is a surjection from X into Y where the range is equal to X
  - such a function is denoted: $F : X \; 7 \; Y$

    $X \; 7 \; Y \; = \; X \; 6 \; Y \; i \; X \; 3 \; Y$

the B language: set theory

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8	>+>>
9	>->>

- *bijective functions*
  - definition: a **partial bijection** is a function from X into Y that is injective and surjective;
  - such a function is denoted: F : X 8 Y

    X 8 Y = X 4 Y i X 6 Y
  - definition: a **total bijection** is a total function from X into Y that is injective and surjective;
  - such a function is denoted: F : X 9 Y

    X 9 Y = X 5 Y i X 7 Y
  - the inverse of a partial bijection is ...

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the B language: set theory

- *sequences*
  - definitions: a sequence of 'elements' belonging to a set *X* is a total function from an interval 1..n into *X* , for $n \in \mathbb{N}$

    the sequence then correspond to the second elements of the maplets of this function, *ordered* by their first elements

    e.g. $\{ 1 \mapsto a, 2 \mapsto b, 3 \mapsto c \}$
  - consequence: function expressions also apply to sequences
- *explicit sequences*
  - [ ] the empty sequence
  - $[x_1,\ldots x_n]$ sequence of *X* defined by enumeration

    e.g. $[a, b, c] = \{1 \mapsto a, 2 \mapsto b, 3 \mapsto c\}$

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the B language: set theory

on paper	at the keyboard
j	<-
k	->

sequence operations

- size (*S*) — length of the sequence *S*
  e.g. size ([ ]) = 0
  size ([a, b, c]) = 3
- first (*S*) — first element of *S* : first (*S*) = *S* (1)
  e.g. first ([a, b, c]) = a
- last (*S*) — last element of *S* : last (*S*) = *S* (size (*S*))
  e.g. last ([a, b, c]) = c
- rev (*S*) — reversal of the sequence *S*
  e.g. rev ([a, b, c]) = [c, b, a]
- *x* k *S* — insertion of *x* before the sequence *S*
  e.g. a k [b, c] = [a, b, c]
- *S* j *x* — insertion of *x* after the sequence *S*
  e.g. [a, b] j c = [a, b, c]

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the B language: set theory

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)	^
q	/|\
w	\\|/

sequence expressions

- S_1) S_2 concatenation of sequences S_1 and S_2

  e.g. [a, b, c]) [c, b, a] = [a, b, c, c, b, a]

- S q n sequence comprising the first n elements of S at most, or S itself when n > size (S)

  e.g. [a, b, c] q 2 = [a, b]

- S w n sequence obtained by removing the first n elements of S

  e.g. [a, b, c] w 2 = [c]

- tail (S) sequence obtained by removing the first element of S

  e.g. tail ([a, b, c]) = [b, c]

- front (S) sequence obtained by removing the last element of S

  e.g. front ([a, b, c]) = [a, b]

the B language: set theory

- *sequence types*
  - seq (X) the set of sequences of X
  - seq1 (X) the set of non-empty sequences of X
  - iseq (X) the set of injective sequences of X
  - iseq1 (X) the set of non-empty injective sequences of X
  - perm (X) the set of bijective sequences of X (permutations on X)

    e.g. perm({a,b,c}) = {[a,b,c], [a,c,b], [b,a,c], [b,c,a], [c,a,b], [c,b,a]}

the B language substitutions

- substitutions represent the transformation of data by programs
- so they change the state of a system
- they concern some list of variables
- substitutions are mathematically defined as *predicate transformers*

the application of a substitution S to a predicate P is noted: $[\ S\ ]\ P$

e.g. $[x := 2]\ (\ x > 1\ )\quad e \quad (\ 2 > 1\ )$

the B language substitutions

- these substitutions are used in the specifications (abstract machine and its possible refinements) and also in the code (implementation) of a module
- in specifications **[Spec]**: a substitution describes abstract properties of operations,
  they may be *non-deterministic*
  e.g. "becomes such that" substitutions
- in implementations **[B0 Code]**: only classical programming language constructs are allowed
  (":=", ";", IF, CASE, WHILE, procedure calls, null statement)

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the B language substitutions

- null substitution [Spec, B0 code]

  skip the variables keep their values (what variable list?)

- "becomes equal to" substitution [Spec, B0 code]

  $v := E$ the value of E is assigned to v

  e.g. $v := 0$

  $x := y + 1$

  $a, b := c, d$

  $f(i) := m$

  $r'b := n$

- "becomes element of" substitution [Spec]

  $v :: X$ an element of X is assigned to v

  e.g. $v :: (1..3)$

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the B language substitutions

- "becomes such that" substitution [Spec]
  - $x :(P_x)$
  - the variable x is assigned a value which satisfies the predicate P_x

    e.g. $x :(x : \text{NAT} \ \& \ x \bmod 3 = 1)$

    (results of dividing n by m, where $n : \mathbb{N}$ and $m : \mathbb{N}_1$)

    $q, r : (q : \mathbb{N} \ \& \ r : \mathbb{N} \ \& \ n = (m * q) + r \ \& \ r < m)$

  - the previous value of x can be referenced in P_x by $x\$0$

    e.g. $x :(x > x\$0)$

the B language substitutions

- *simultaneous substitutions* [Spec]
  - $S_1 \; || \; S_2$
    applies the substitutions S_1 and S_2 simultaneously
    the variables modified in S_1 and S_2 must be *distinct*
    e.g. $x := 1 \; || \; y := 2$
    $x := y \; || \; y := x$
- *sequential substitutions* [B0 code]
  - $S_1 \; ; \; S_2$
    applies the substitution S_1 *and then* the substitution S_2
    e.g. $x := 1 \; ; \; y := 2$
    $x := y \; ; \; y := x + 1$

the B language substitutions

- *"BEGIN" substitution (block substitution)* [Spec, B0 code]
  - BEGIN *S* END used to parenthesize substitutions

    e.g. BEGIN $x := y \;||\; y := x$ END
    BEGIN $x := y \;;\; y := x + 1$ END

- *"VAR" substitution (block of local variables)* [B0 code]
  - VAR $v_1, \ldots, v_n$ IN *S* END

    introduction of local variables $v_1, \ldots v_n$ that may be used in substitution *S*

    e.g. VAR t IN $t := x;\; x := y;\; y := t$ END

the B language substitutions

- *pre-condition* [Spec]

  → PRE P THEN S END

  a substitution that may only be used when the predicate P holds
  used to specify the properties that have to hold when calling an operation

  e.g. PRE $x : NAT_1$ THEN $x := x - 1$ END

- *assertion* [Spec, B0 code]

  → ASSERT P THEN S END

  similar to a precondition, but used to simplify the proof,
  by factorizing a property

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the B language substitutions

- *"ANY" substitution* [Spec]
  - ANY x WHERE P_x THEN S END

    apply the substitution S in which, the variables x that satisfy P, can be used (in read only)

    e.g. ANY x WHERE $x \in NAT \wedge x < 10$ THEN $y := x + 1$ END
  - Note: the "ANY" substitution is very versatile

skip	ANY x WHERE $x = y$ THEN $y := x$ END
$y :=$ a	ANY x WHERE $x =$ a THEN $y := x$ END
$y :\in E$	ANY x WHERE $x \in$ E THEN $y := x$ END
$y : (P_Y)$	ANY x WHERE P_x THEN $y := x$ END

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the B language substitutions

- *"CHOICE" substitution* [Spec]
  - → CHOICE S_1 OR S_2 … OR S_n END
    apply one of the substitutions S_1, S_2, … , S_n
    e.g. CHOICE $x := x + 1$ OR $x := x - 1$ OR skip END
- *"SELECT" substitution* [Spec]
  - → SELECT P_1 THEN S_1 WHEN P_2 THEN S_2 … ELSE S_n END
    defines several branches of substitutions S_i "guarded" by P_i
  - a substitution may be applied if its guard holds
  - if no guard holds, then the ELSE substitution is applied

    e.g. SELECT $x > 10$ THEN $x := x - 10$
    WHEN $x < 10$ & $x > 0$ THEN $x := 2 * (x - 10)$
    ELSE $x := x - 1$
    END

the B language substitutions

- *"IF" substitution* [Spec, B0 Code]

  IF P_1 THEN S_1 ELSIF P_2 THEN S_2 … ELSE S END

  the substitution applied is:

  - S_i if P_i holds **and** the previous predicates do not hold
  - S if no predicate P_i holds (by default S is skip)

  e.g.

```
IF  x > 10  THEN
      x := x - 10
ELSIF  x = 0  THEN
      x := x + 1
ELSE
      x := 1
END
```

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the B language substitutions

- *"CASE" substitution* [Spec, B0 Code]

  CASE V OF EITHER V_1 THEN S_1 OR V_2 THEN S_2 ... ELSE S_n END END

  the substitution applied is:

  - S_i if V belongs to the list of literals V_i (the V_j have to be distinct)
  - S otherwise (by default S is skip)

```
e.g. CASE x OF
         EITHER 0, 1, 2 THEN x := x + 1
         OR 3, 4 THEN x := x - 1
         OR 10 THEN skip
         ELSE x := x + 10
         END
     END
```

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the B language substitutions

- *"WHILE" substitution* [B0 Code]
  - → WHILE *P* DO *S* INVARIANT *I* VARIANT *V* END
    - while loop, or iterative behaviour:
      while the predicate *P* holds, the "loop body" *S* is applied
    - the negation of *P* is the "exit condition" from the loop
    - the loop INVARIANT parts gives the properties that hold just before the loop, and after every iteration. It should give a recurrence relation on the variables modified inside the loop
    - the VARIANT clause defines a *decreasing positive* expression, in order to prove that the number of iterations is finite, and so that the loop terminates

the B language substitutions

on paper	at the keyboard
c	<--

- *operation calls* [Spec, B0 Code]
  - ➔ application of the substitution specified for the operation *op*, with replacement of its formal parameters by the actual parameters "call by value"

    e.g. op1

    op2 (*y* - 1)

    x c op3

    x, *y* c op4 (*x* + 1, TRUE)

the B language
data typing

- *data typing principles*
  - every data item must be typed before being used
  - types within the B language are based on the set theory
  - predicates, expressions and substitutions have to respect *typing rules*
  - such rules avoid obviously meaningless constructs ("don't mix apples and oranges")

    e.g. 2 = TRUE

- *Definition*
  - *the type of a data item is the largest B set to which it belongs*

the B language
data typing

- *B types*

  every type is described in terms of *basic types* and *type constructors*

  ➔ the basic types are

  - BOOL
  - Z
  - fixed or enumerated sets (see the **SETS** clause)

    e.g. TRUE : BOOL

    2 : Z

  ➔ the type constructors are

  - subsets P (T)
  - Cartesian products T_1 * T_2

    e.g. {1, 3, 5} : P (Z)

    (0 m FALSE) : Z * BOOL

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the B language
data typing

- *how are data items typed?*
  - in general data items are typed by *typing predicates*, which are particular predicates of the form

    untyped_data_item typing_operator typed_expression

    where the *typing_operators* are '=', ':' et '('

    e.g. x : 1..10 & y : BOOL 3 INT & $z = x + 1$ & S (INT

    such typing predicates must be at the highest syntactic level within a conjunction list
  - local variables and result parameters of an operation are instead typed by *typing substitutions*

    e.g. VAR t IN ... $t := x + 1$; ... END

    r c op(p) = PRE p : NAT_1 THEN ... $r := p - 1$ ... END

the B language

- *B components (reminder)*
  - **static aspect**
    - definition of the subsystem state space: *sets*, *constants*, *variables*
    - definition of static properties for its state variables: *invariant*
  - **dynamic aspect**
    - definition of the initialisation phase (for the state variables)
    - definition of operations for querying or modifying the state
  - **proof obligations**
    - the static properties must be mutually consistent
    - they must be *established* by the initialisation
    - they must be *preserved* by all operations

the B language
form of components

- *static aspect*
  - set definitions (SETS clause)
  - constant definitions (CONSTANTS, PROPERTIES clauses)
  - variable definitions (VARIABLES, INVARIANT clauses)
  - set and constant values (VALUES clause)
  - machines with parameters (CONSTRAINTS clause)
  - textual abbreviations (DEFINITIONS clause)
  - supplementary assertions (ASSERTIONS clause)
- *dynamic aspect*
  - initialisation phase (INITIALISATION clause)
  - operation definitions (OPERATIONS clause)

form of B components: static aspect

- *set definitions*

SETS S_1; … ; S_n

this clause introduces new base types into a component

→ a fixed set is defined by its name X_i

e.g. SETS *STUDENTS*

its *value* is not yet defined, it will be given in the implementation

eventually its value is a non-empty implementable interval

→ an enumerated set is defined by its name and the list of its elements: $X_i = \{x_1, \dots x_m\}$

e.g. SETS *COLOR* = { *Red, Green, Blue*}

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form of B components: static aspect

- *constant definitions*
  - ABSTRACT_CONSTANTS $x_1,...,x_n$
    (CONCRETE_)CONSTANTS $x_1,...,x_n$
  - these clauses introduce new constants into a component
    a constant may be read but not modified
  - a concrete constant is directly implementable (scalar, interval, array),
    it is automatically preserved through refinement,
    it has to be *valued* in the implementation
  - an abstract constant is a constant of any arbitrary type,
    it is not automatically preserved through the refinement,
    it is not allowed in implementations

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form of B components: static aspect

- *constant definitions*

  PROPERTIES $P_{x1,\dots,xn}$

  the PROPERTIES clause defines the types and other properties of the constants

  e.g. CONSTANTS *c1*, *c2*
  ABSTRACT_CONSTANTS *c3*

  PROPERTIES
  $c1 : 0..10$ &
  $c2 : 0..10$ &
  $c1 + c2 < 15$ &
  c3 : N 3 0..15

form of B components: static aspect

- *variable definitions*
  - (ABSTRACT_)VARIABLES $v_1,\ldots,v_n$
    CONCRETE_VARIABLES $v_1,\ldots,v_n$
  - these clauses introduce new variables into a component
  - an abstract variable is a data item of any arbitrary type, it is not automatically preserved through the refinement, it is not allowed in implementations
  - a concrete variable is a variable directly implementable (scalar or array), it is automatically preserved through refinement

form of B components: static aspect

- *variable definitions*
  - ➔ INVARIANT $P_{v1,\ldots,vn}$

    e.g. VARIABLES

    A, B

    INVARIANT

    $A \subseteq T \land$

    $B \subseteq T \land$

    $\mathrm{card}(A \cap B) = 1$
  - ➔ the INVARIANT clause defines the types and other properties of the variables
  - ➔ after the module initialisation, these properties remain invariant after any operation call

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form of B components: static aspect

- *example*

```
MACHINE
        Register                                } machine name
SETS
        STUDENTS                                } fixed set
CONCRETE_CONSTANTS
        max_students
PROPERTIES
        max_students : NAT                      } constant type
ABSTRACT_VARIABLES
        Enrolled
INVARIANT
        Enrolled ( STUDENTS &                   } variable type,
        card (Enrolled) < max_students          } and properties
...
END
```

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form of B components: static aspect

- *values of fixed sets and concrete constants*
  - → *e.g.* VALUES

    STUDENTS = 0..255;

    max_students = 255;

    transfer = {0 m FALSE, 1 m TRUE, 2 m FALSE};

    default_grade = (0..255) * {0}
  - → the VALUES clause is only allowed in implementation
  - → it should give a value to every
    - fixed sets of the B module
    - concrete constants of the B module
  - → fixed sets are eventually valued with implementable intervals

form of B components: static aspect

- *textual abbreviations*
  - the DEFINITIONS clause defines textual abbreviations, which may then be used as expressions in the rest of the current component (similar to #define in C language)
  - definitions may have parameters and may be factorized in definition files

    e.g. DEFINITIONS

    ```
    NMAX == 255 ;
    NMAXm1 == NMAX - 1 ;
    no(b) == bool(b = FALSE) ;
    "mydef.def"
    ```

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form of B components: dynamic aspect

- *initialisation phase*

  INITIALISATION *S*

  this clause defines the initial values of the component variables

  initialisation has to *establish* the invariant

  ex. : ABSTRACT_VARIABLES

  Enrolled

  INVARIANT

  Enrolled (*STUDENTS*

  INITIALISATION

  Enrolled := 0

form of B components: dynamic aspect

- *operation definitions*
  - the OPERATIONS clause defines operations (B procedures or functions)
  - each operation defined in an abstract machine has to be redefined in the refinements of the abstract machine
  - it is not possible to introduce new operations within refinements (or implementations)
  - operations may have input and output parameters defined in the operation header, that are implementable
  - properties of input parameters that have to be proved when calling the operation are defined in the precondition (useful only for abstract machines)
  - output parameters are typed in the substitution of the operation specification
  - an operation is a substitution that defines how **all** of the component variables and the output parameters are modified

form of B components: dynamic aspect

- *operation definitions*
  - ➔ syntax of operations

$$op \;=\; S$$
$$op\,(\,p_1,\ldots,p_n\,) \;=\; S$$
$$r_1,\ldots,r_m \;\mathrm{c}\; op \;=\; S$$
$$r_1,\ldots,r_m \;\mathrm{c}\; op\,(\,p_1,\ldots,p_n\,) \;=\; S$$

  - ➔ operations that change some state variables are *modifying operations* (otherwise *querying operations* or *read-only*)
  - ➔ operations have to *preserve* the invariant
  - ➔ operation refinements or implementations have to *be consistent* with their specifications

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form of B components: dynamic aspect

- *example (version 2 completed)*

```
MACHINE
        Register
SETS
         STUDENTS
ABSTRACT_VARIABLES
        Enrolled
INVARIANT
        Enrolled ( STUDENTS

/* dynamic */

INITIALISATION
        Enrolled :=  0
...
```

```
...
OPERATIONS
        n c num_ enrolled  =
                n :=  card (Enrolled) ;
        b c is_ enrolled ( s ) =
                PRE  s : STUDENTS
                THEN  b :=  bool ( s : Enrolled )
                END ;
        enrol_student ( s ) =
                PRE  s : STUDENTS – Enrolled
                THEN Enrolled :=  Enrolled u {s}
                END ;
        withdraw_student ( s ) =
                PRE  s : Enrolled
                THEN Enrolled :=  Enrolled - {s}
                END
END
```

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form of B components: dynamic aspect

- *local operation*
  - → a new feature to avoid too much levels of module in a project
  - → to make the proof of implementations easier

  - → they may be defined only in implementations
  - → they may be called from implementation operations (global or local)
  - → they are specified in the LOCAL_OPERATIONS clause
    (abstract specifications on the the visible variables)
  - → their invariant is made up by the typing of the concrete variables
  - → they are implemented in the OPERATIONS clause
    (like global operations)

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form of B components: dynamic aspect

- *local operation example*

```
IMPLEMENTATION
...
LOCAL_OPERATIONS
        r c GetMax(x, y) =
                PRE x : INT & y : INT THEN
                r := max ({x, y})
                END
OPERATIONS
        r c GetMax(x, y) =
                IF x < y THEN r := y
                ELSE r := x
                END ;
        …
        op =
                …
                v c GetMax(z, 10);
                …
END
```

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the B language

- *modular construction*
  - the B language supports modularity: breaking down large sub-systems, building up small sub-systems into larger ones
- *modular mechanisms*
  - importing abstract machines at implementation level (IMPORTS clause)
  - read-only visibility of other abstract machines (SEES clause)

the B language: decomposition

- *importing of other machines*

  e.g. IMPLEMENTATION
  A_i
  REFINES
  A
  IMPORTS
  B, C

  - the IMPORTS clause may only appear in implementations
  - an implementation *imports* other abstract machines in order to implement data and operations with lower level machines: this is the main breaking down mechanism in B
  - variables of an imported machine may be modified in the implementation by operation calls

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the B language: decomposition

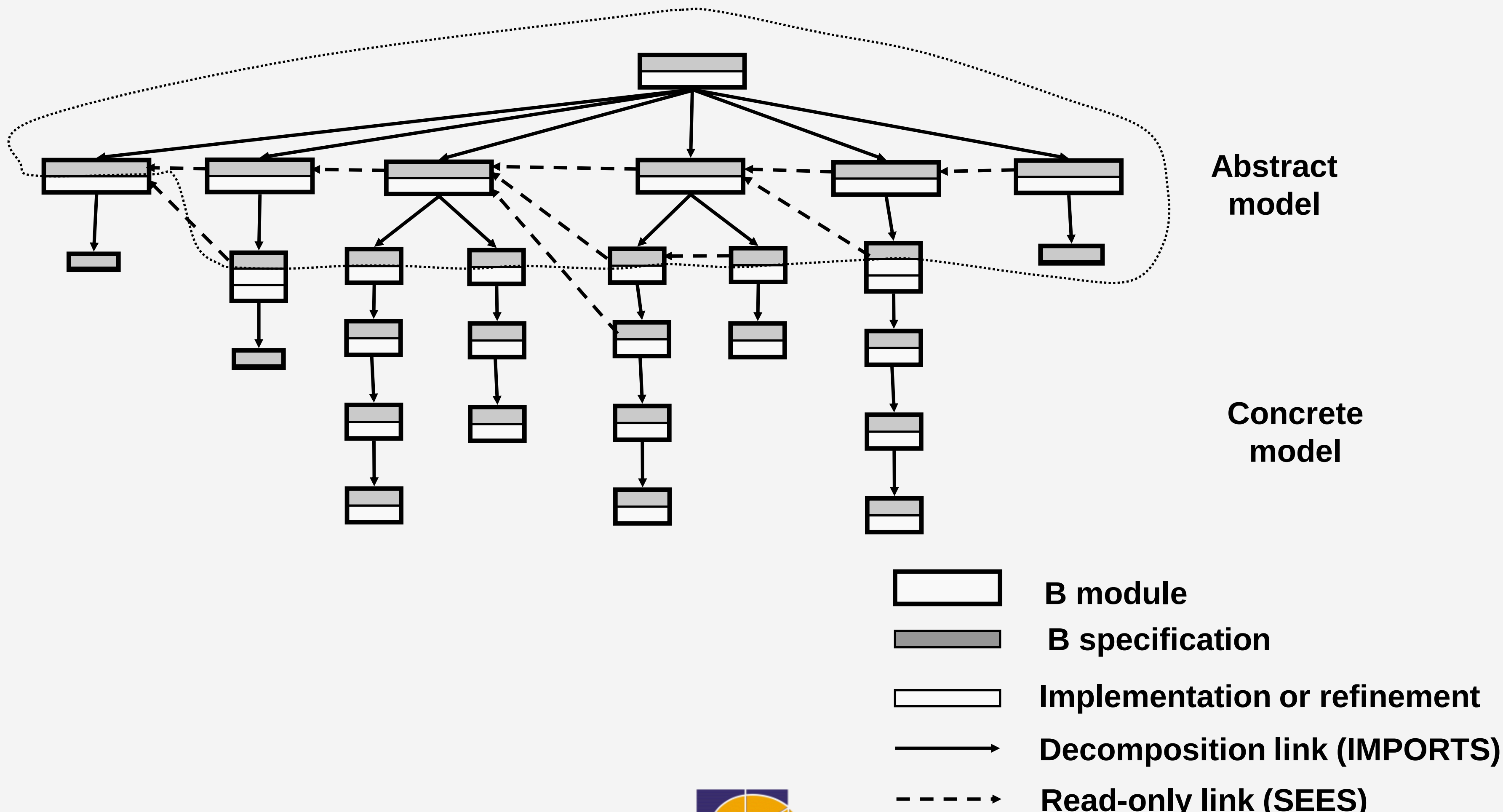
- *visibility of other machines*

  e.g. MACHINE
  A
  SEES
  B, *C*

  → when a component *sees* an abstract machine *M*, the data of *M* may be accessed in read-only, modification operations of *M* can not be called

  → the SEES link is not transitive

B-Software links



the B language
B0

- *B0 is the part of B that may be translated*
  - the name of the abstract machine and the links in the implementation
  - the concrete data of the module: sets, concrete constants, concrete variables, the valuation of sets and concrete constants
  - implementation initialisation
  - operations: operation parameters and the substitutions of implementation operations
- *the data must be concrete*
  - scalar data: INT, BOOL, sets (fixed and enumerated sets)
  - sub-intervals of INT
  - implementable arrays

B Training Sessions

- *Level I: understanding B*
  - overview of the B method and the B language
  - tutorials and practical with Atelier B: specification, writing a program that is consistent with its specification, notion of proof
- *Level II: applying B*
  - advanced notions of the B method, B for systems
  - tutorials and practical with Atelier B: building a large program, Proof Obligations
- *Level III: proving*
  - learning to use Atelier B to prove a project
  - practical: automatic and interactive proof, proof tools, user rules

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